# Age-period-cohort models

Niels Keiding
January 2007

# Age-period-cohort models

| | | |
|---|---|---|
| $t$ | time at birth | cohort |
| $x$ | age | age |
| $\tau = t + x$ | current time | period |

Death intensity $\mu(\tau, x)$:

$$\mu(\tau, x) = \frac{1}{h} P \text{ (individual dies in } (\tau, \tau + h) \times (x, x + h) \mid \text{ alive at } (\tau, x))$$

Assume $\mu(\tau, x)$ constant across $1 \times 1$ year groups

$$\mu(\tau, x) = \mu\big([\tau], [x]\big) =: \mu_{\tau,x}$$

Age-period-cohort model

$$\mu_{\tau,x} = \varepsilon_\tau \varphi_x \kappa_t \qquad \tau = t + x$$

# Identification problem

$$\mu_{\tau,x} = \varepsilon_\tau \varphi_x \kappa_t \qquad t = \tau - x$$

$$\mu_{\tau,x} \qquad \tau = 1,\cdots,T \qquad x = 1,\cdots,A$$

$$\Rightarrow \quad t = \tau - x = -A+1,\cdots,T-1$$

so

$T$ $\varepsilon_\tau$'s

$A$ $\varphi_x$'s

$A+T-1$ $\kappa_t$'s

$2A+2T-1$ parameters

*but* 3 independent constraints given by arbitrary choices of $\xi,\eta,\zeta$

$$\varepsilon_\tau \varphi_x \kappa_t = \left(\varepsilon_\tau \xi \eta^{\tau}\right)\left(\varphi_x \xi^{-1} \eta^{-x} \zeta\right)\left(\kappa_{\tau-x} \eta^{-(\tau-x)} \zeta^{-1}\right)$$

$2A+2T-4$ parameters

Tricky problem: the linear effect $\log\eta$ unidentifiable

# The Danish life expectancy debate

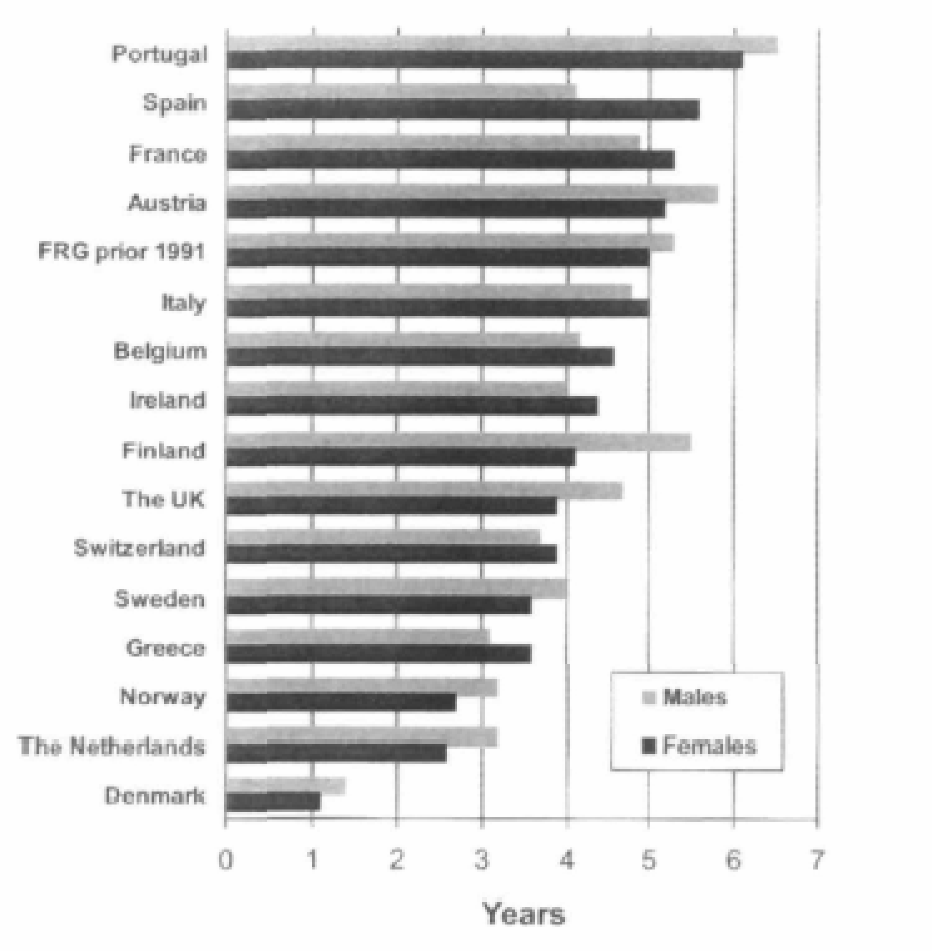
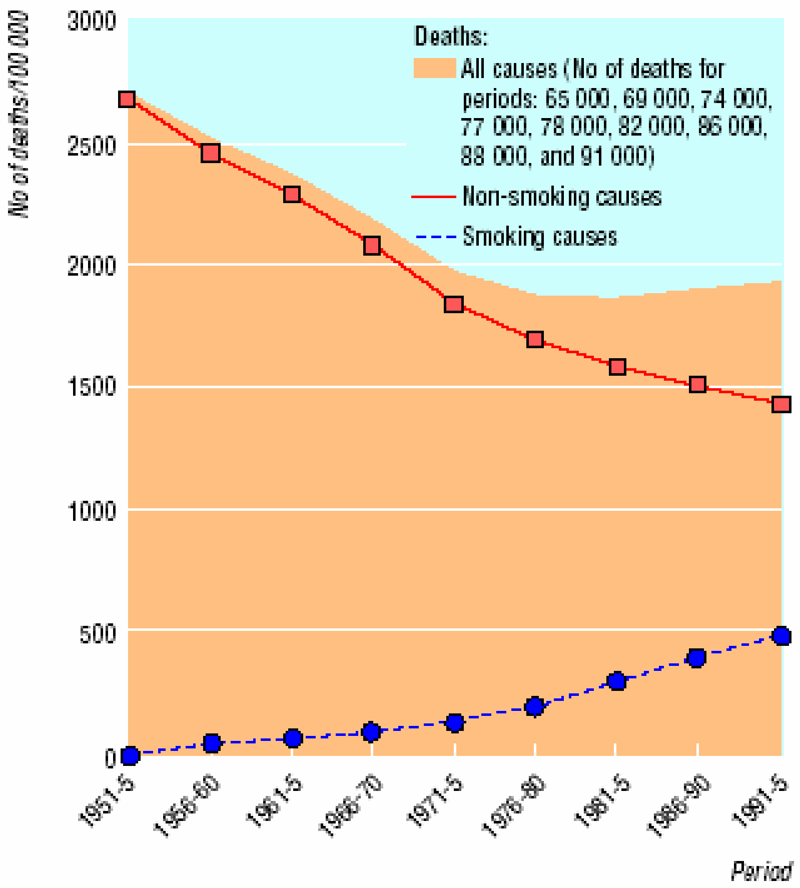



Figure 1 Increase in life expectancy (years) in a 20 year period (ca. 1975–1994) in 16 Western European countries by sex

**Mortality and life expectancy in Denmark and in other European countries.**
K Juel; P Bjerregaard; M Madsen
*European Journal of Public Health;* Jun 2000; 10, 2; ABI/INFORM Global
pg. 93




Trends in age standardised mortality (per 100 000, world standard) for deaths attributed to smoking and to causes other than smoking among women aged 55-84 in Denmark, 1951-95

# Increased mortality among Danish women: population based register study

Knud Juel

*BMJ* 2000;321;349-350

# Statistics: “Poisson” regression

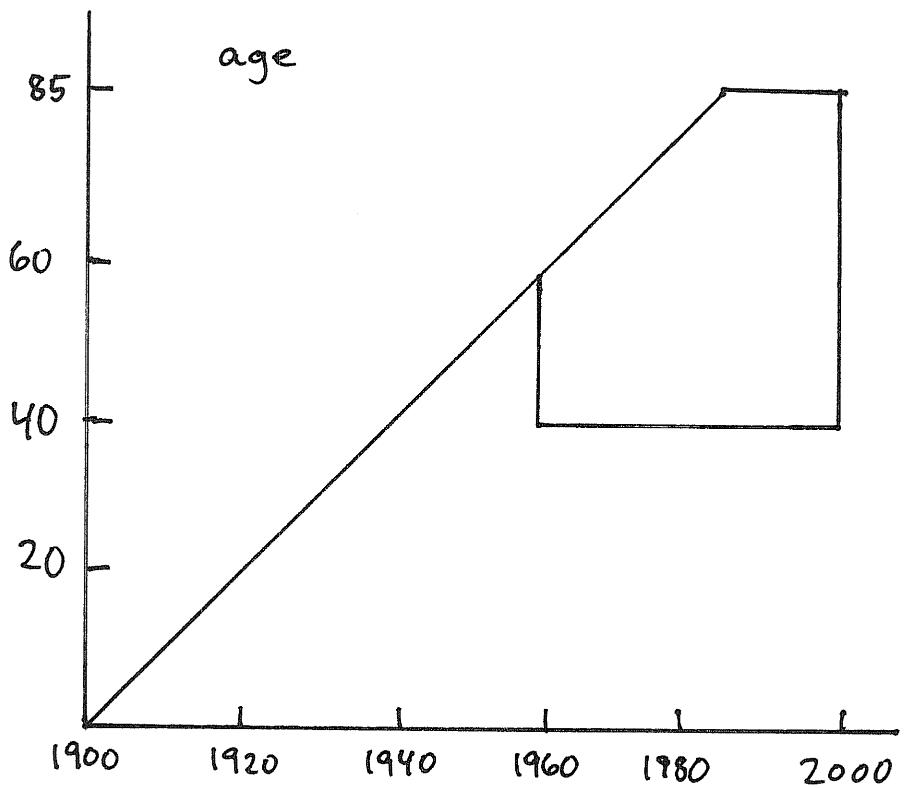
Standard goodness-of-fit measures cannot be expected to agree. Overdispersion models?

Usually: graphical checks

# Mortality of Danish women

R. Jacobsen, N. Keiding, E. Lynge (2002). *J.Epidemiol.Comm.Health* **56**, 205-208.

R. Jacobsen, A. Jensen, N. Keiding, E. Lynge (2001). *Lancet* **358**, 75.

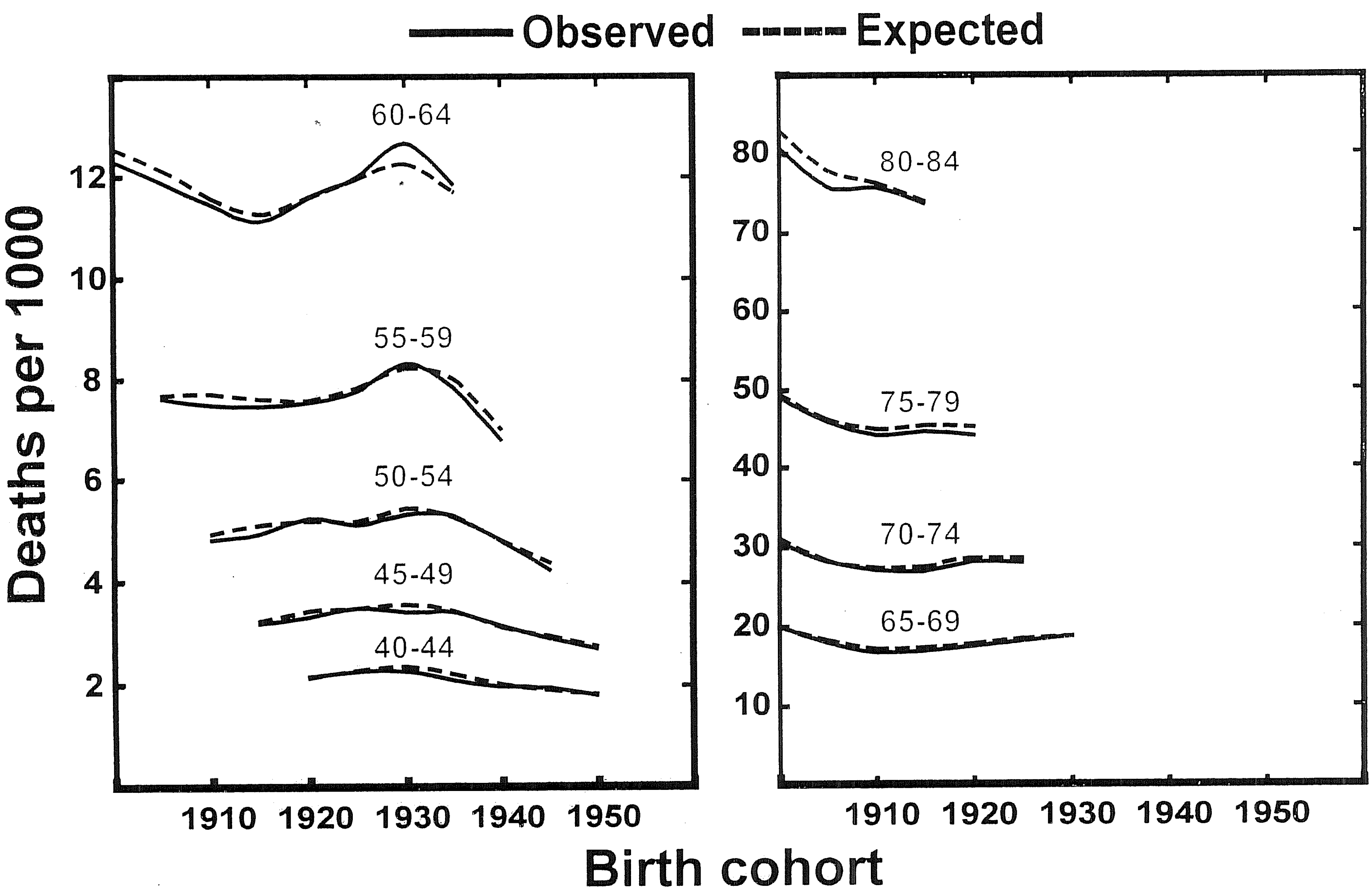



All Danish women.

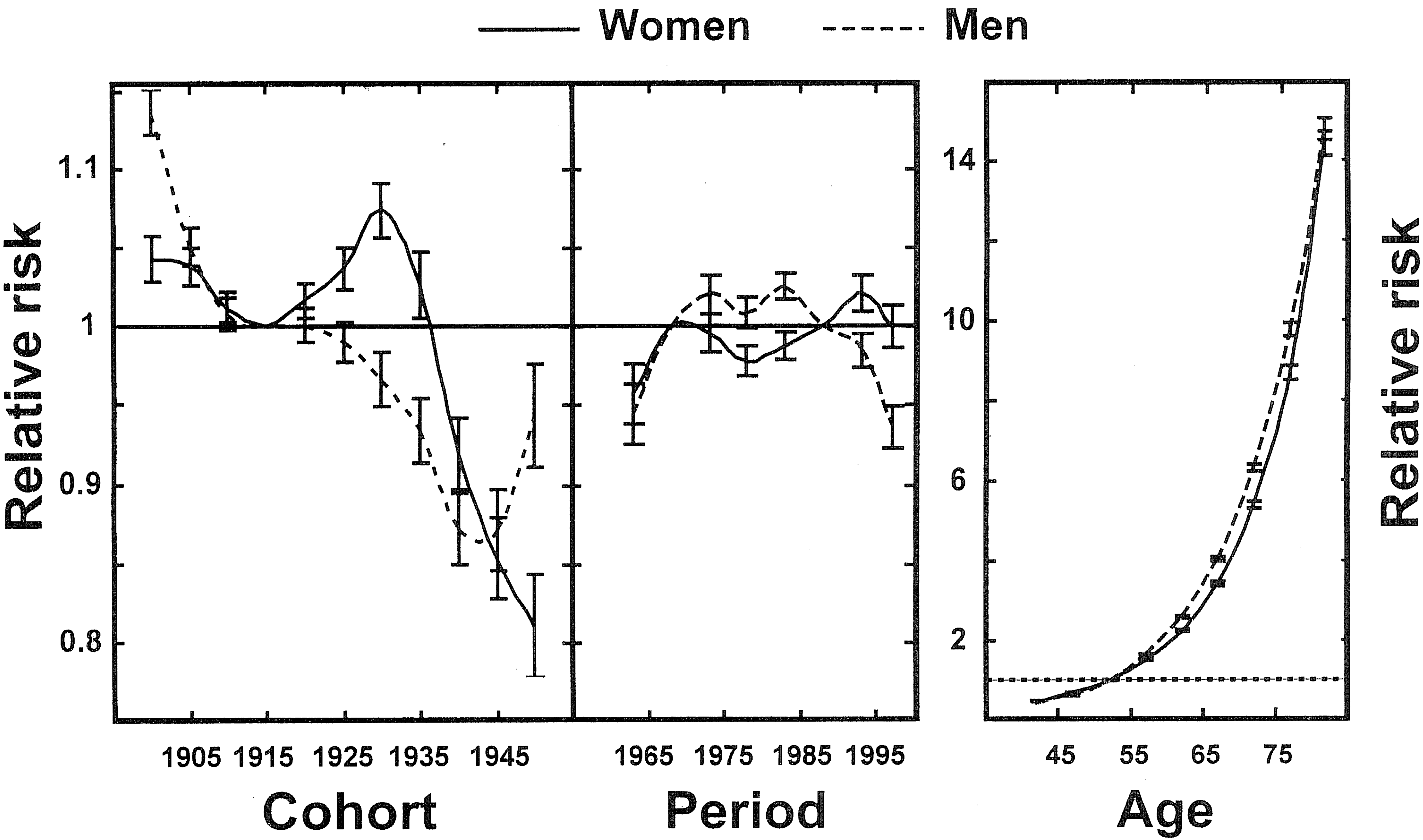
Deaths during 1960-99, ages 40-84 years, cohorts later than 1900.

Age-period-cohort model fits better than usual with deviance 67 on 36 d.f.

# Female Mortality



# Mortality


Women
Men
Relative risk
1.1
1
0.9
0.8
1905
1915
1925
1935
1945
Cohort
1965
1975
1985
1995
Period
14
10
6
2
45
55
65
75
Age
Relative risk

# Results

Men: linear period effects, linear cohort effects

Women: linear period effects, **nonlinear** cohort effects 1925-1945

# The unidentifiable linear effect

Illustrate through sensitivity analysis-type variation of the partition of the linear "drift" term $\delta$ between period and age + cohort

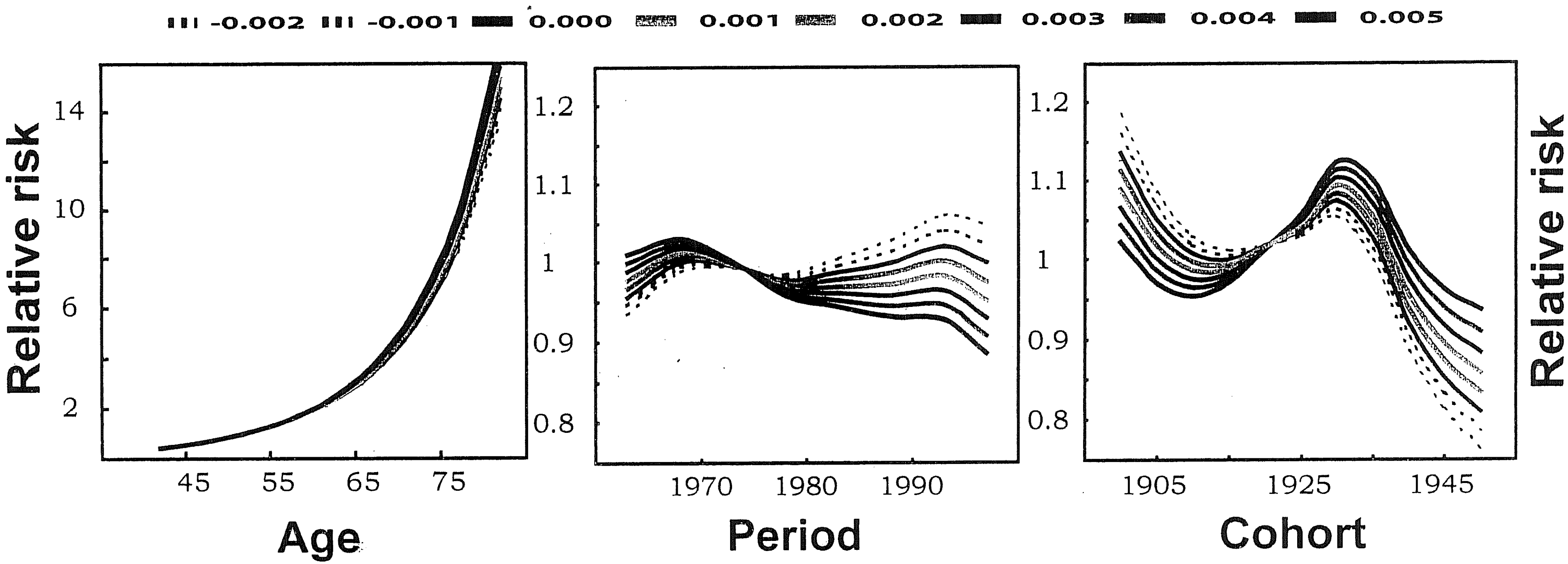
$$\log \mu_{\tau,x} = \log \varepsilon_\tau \varphi_x \kappa_t$$

$$= \alpha_\tau + \beta_x + \gamma_t$$

$$= (\alpha_\tau + \tau\delta) + (\beta_x - x\delta) + (\gamma_t - t\delta)$$

# Female Mortality

## Partition of drift: period vs. cohort + age



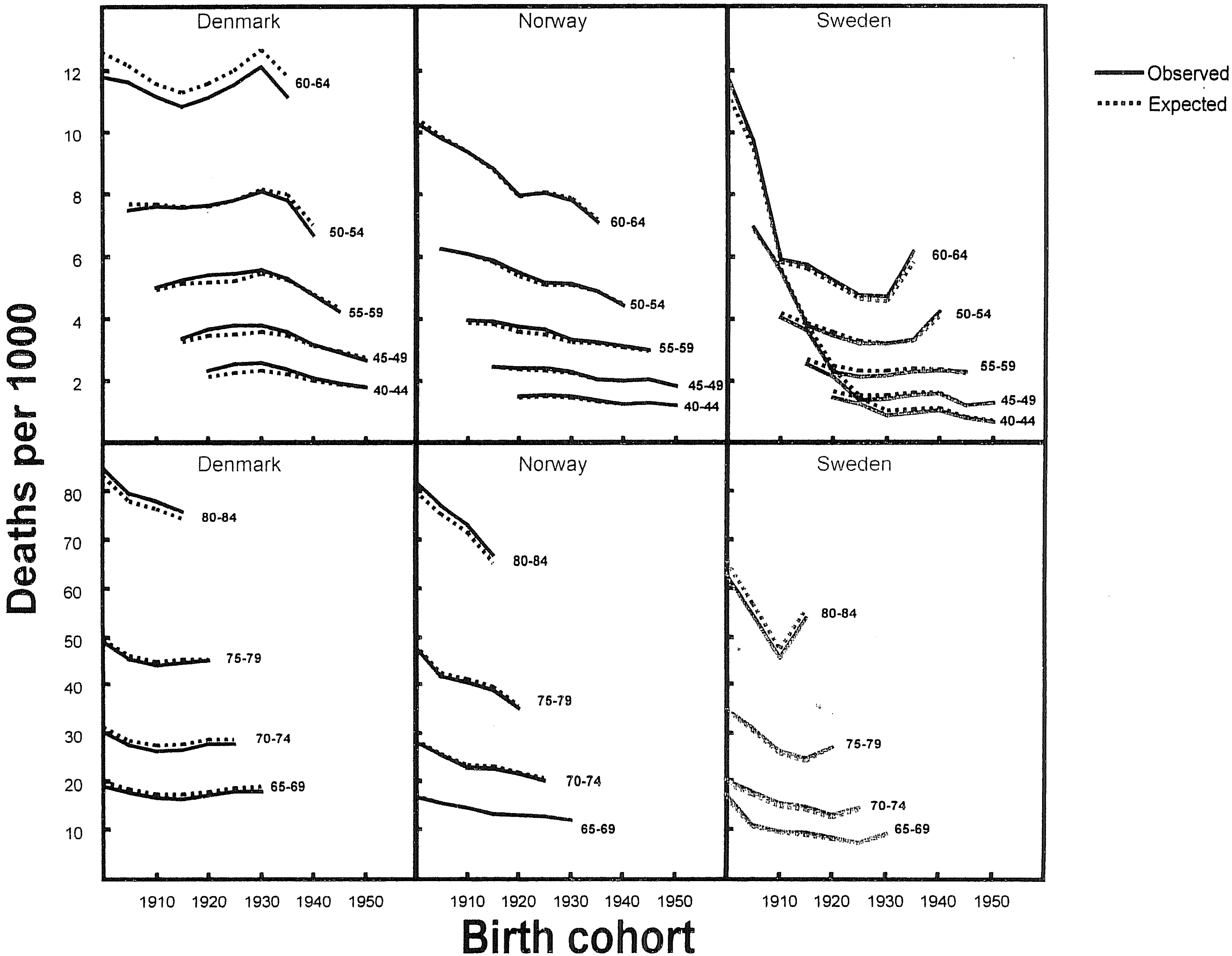
# Women in Denmark, Norway, Sweden

R. Jacobsen, M. von Euler, M. Osler, E. Lynge, N. Keiding (2004). *Eur.J.Epidemiol.* **19**, 117-121.

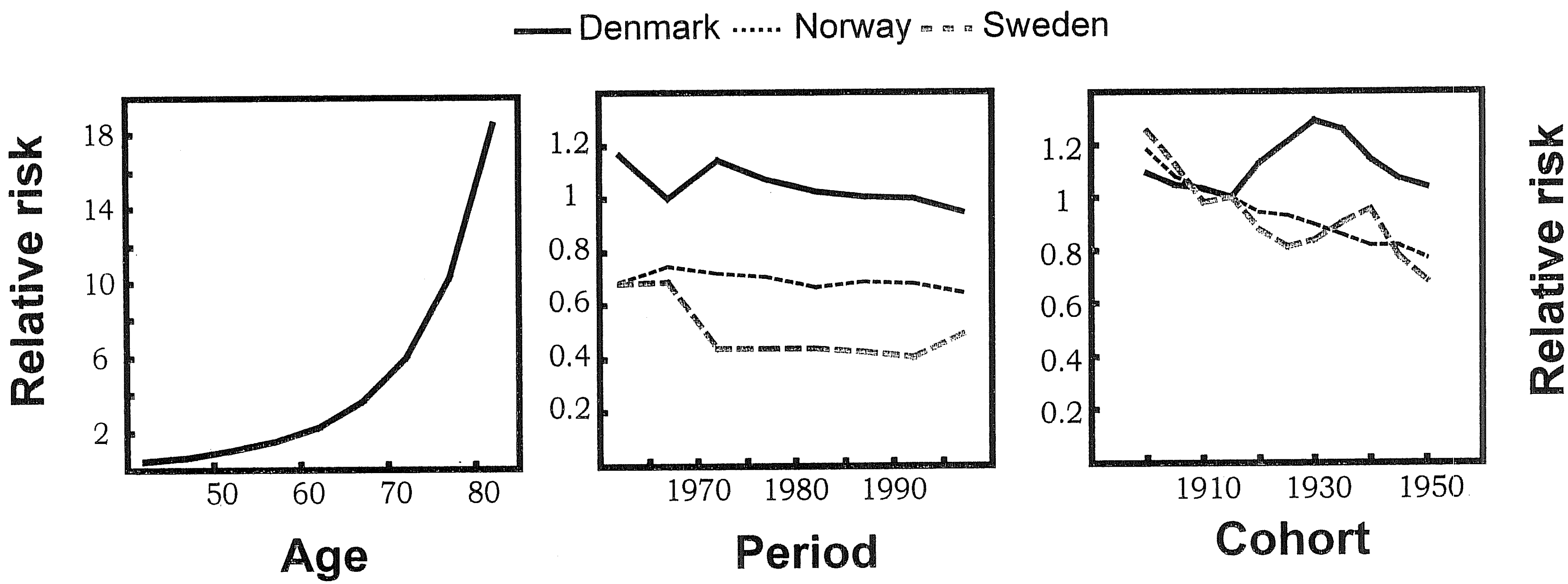
Assume common underlying age effect so for country $n$

$$\log \mu^n(t,a) = \alpha_t^n + \beta_a + \gamma_c^n .$$

# Female Mortality



# Female Mortality



# Women in Denmark, Norway, Sweden: results

| | Period | Cohort | Level |
|---|---|---|---|
| Denmark | linear | nonlinear | high |
| Norway | linear | linear | lower |
| Sweden | somewhat nonlinear in 1970s | linear | lowest |